AMERICAN BOND OPTION PRICING OFFERED BY RISKWORX

RISKWORX

South African bonds trade on yield, and options are struck on yield, but the volatility quoted or used in calculation of the bond option price is typically an (all-in-price) volatility.

Options are typically American, and this introduces the necessity to abandon analytic approaches and resort to tree methods. Even for European options, it is our opinion that tree methods are superior. This is because the volatility of a bond demonstrates a significant term structure (because of the pull to par effect) whereas the Black formula we have seen is assuming a constant volatility.

In (Quant Financial Research 1998), the suggestion is made to value American options by simply making an intrinsic correction to the European formula. This is a reasonable suggestion given the context of that document - risk management - but it is completely absurd to use this model for trading or for building into trading systems[1]. Moreover, the volatility used there is a clean volatility, which is not in compliance with the aforementioned practice of the South African market trading on all-in volatility (and occasionally, yield volatility).

Even in the case where the volatility used is a yield volatility, the analysis to follow is still pertinent. Some attempts have been made to build trees of yield, using the yield volatility. This approach does capture the pull to par effect of bonds. However, it is impossible to build trees based on yield while assuring any or all of no arbitrage pricing, the preservation of put-call parity, smooth Greek calculations, and reasonable computational time.

Our intention first is to model the price volatility as a function of time through the life of the option. This is essentially an acknowledgement of the pull to par effect - the price volatility is a decreasing function of time. It will be assumed that the price volatility so quoted is the terminal volatility i.e. the volatility at the expiry of the option. It has to have been higher than that earlier in the option because of the pull to par. Our assumption will be that the yield volatility is constant throughout the life of the option. We will model the relevant price volatility as a function of that assumption.

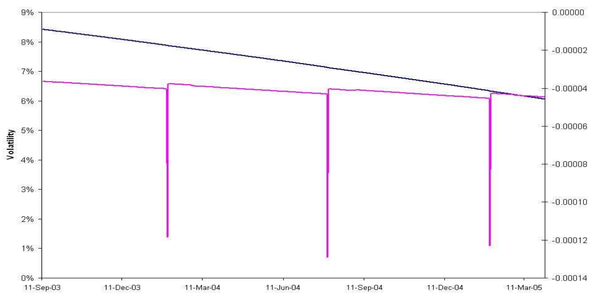

FIGURE 1. Term structure of squeaky clean price volatility, and its derivative

Following broadly the methodology of (Derman, Kani & Chriss 1996), we build a clean price trinomial tree for the bond, following the following steps:

1.1. **The first vanilla tree.** Use zero interest rates and constant volatility, and build a vanilla trinomial

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[1]as some, not to be named, international trading system vendors have done in patching their systems for the South African market. In general, these patches are very poor. Unfortunately, the cited document seems to be used as a benchmark - and their first call of defence - by such vendors without trying to appreciate its intended use.

tree for the clean price. We do this by simply combining into one, two steps of a Cox, Ross, Rubinstein (Cox, Ross & Rubinstein 1979) tree.

1.2. **The term structure of volatility.** For the term structure of volatility: make the mesh size irregular to better accommodate these variations of the local volatilities with time. See (Derman et al. 1996, Appendix B). Thus we introduce the notion of scaled time.

1.3. **Adjust for forward growth.** Now grow the entire lattice along the forward curve by multiplying all node prices at time t_i by $\frac{\mathbb{C}(t_i)}{\mathbb{C}(t_0)}$. This is very effective at modelling the pull to par effect.

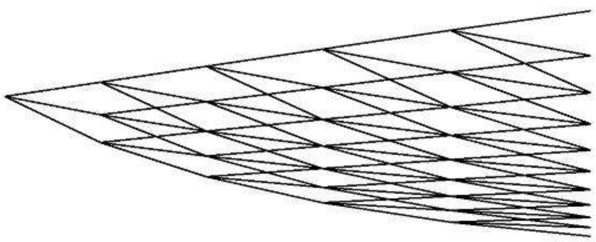

FIGURE 2. The trinomial tree with time adjustment, and drift

Option pricing is now completely standard. For European options, we simply induct backwards through the tree. For American options, we do the same, but check for early exercise at each point. See (Hull 2002) or (Haug 1998) for this mechanism.

For the Greeks we again apply some standard methods, see (Hull 2002, §18.2), for example.

- Delta and gamma can be calculated relative to yields or relative to prices. In either case we use the extra envelope of the tree that we have built. Thus at the initialisation date there are three nodes: the spot node, used for valuation, an up node and a down node, which are used for the greeks.
- Vega and Rho are found by rebuilding the tree at twitched (up and down) volatilities and risk free rates respectively, and then central differences are found . Thus, there is greater computational time taken for these greeks, and slightly less numerical stability.
- Theta is found by annualising the difference in value between the option at the base of the tree and the option at the central node one step into the tree.

All the Greeks are quite stable with respect to the choice of number of nodes, after the usual instability for a very small number of nodes. It seems that 20 time nodes is adequate for all purposes, and so this value has been set as a default.

The delta is stable with respect to market moves. This is a very important criterion for hedging purposes; some tree models are known to fail this property.

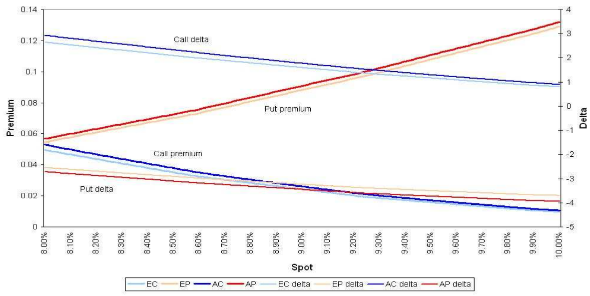

FIGURE 3. Premiums and deltas over a 200bp strike range (spot at centre)

2. The PriceWorX software

The software written by RiskWorX allows for pricing of American or European bond options on any of the bonds listed at the South African Bond Exchange. Valuation of options as well as all greeks and hedge parameters are available.

The software also allows for stressing of any of the following variables:

- the spot yield to maturity of the bond;
- volatility of the bond;
- the risk free rate(s).

Graphical features allow for the visualisation of the stress graph (stressing one of the variables) or stress

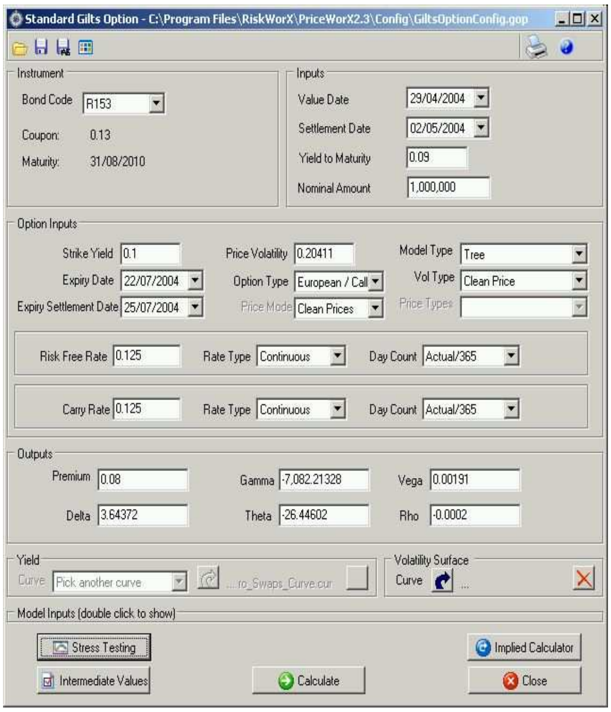

FIGURE 4. Input screen for pricing and the numeric results

surface (stressing two of the variables simultaneously, see Figure 5) or simultaneous stress graphs (the stress surface collapsed into a number of stress graphs, see Figure 6).

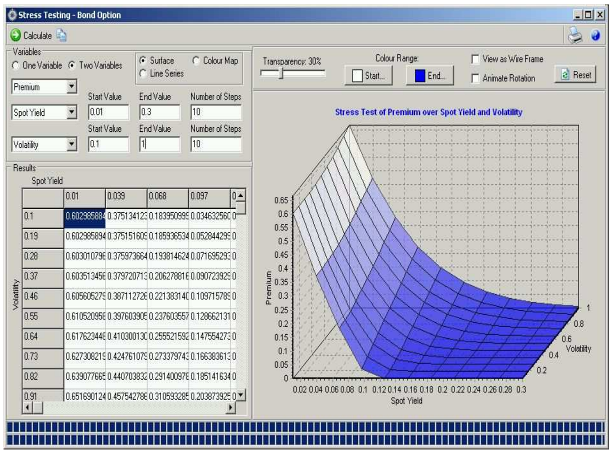

FIGURE 5. The surface of prices for varying yield to maturity and volatilities

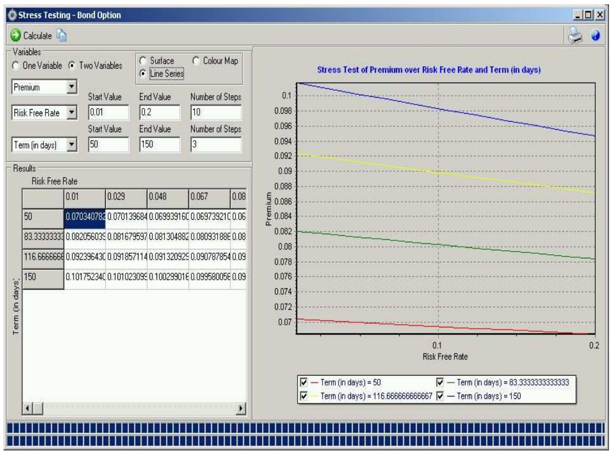

FIGURE 6. The stress graphs for varying risk free rate

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*http://www.bondex.co.za/trading/index.html

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