VANILLA EQUITY OPTION PRICING OFFERED BY RISKWORX

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Typically European equity options are priced using the Black-Scholes model [1] or that model adjusted for dividends by calculating a continuous dividend yield. This has the effect of spreading the dividend payment throughout the life of the option. In the case of several dividend payments, this is a satisfactory solution, for example, where the option is on an index (where the index is paying out several dividends, spread out through the period of optionality).

Thus, for European equity options where the underlying has no or several dividends, we will use the Black-Scholes formula. For American equity options with the underlying having no or several dividends, we may argue similarly. Here the approximation of Barone-Adesi and Whaley [2] is popular, but we prefer the method of Bjerksund and Stensland [3], [4] as it is computationally far superior, and has been shown to be more accurate in long dated options. [4] is a recent improvement over [3].

Another standard approach (for the European case) is to reduce the stock value by the present value of dividends (the escrowed dividend method), or to increase the strike by the future value of dividends. Both are unsatisfactory approaches as they affect the stochastic process on the equity fairly significantly. See [5], [6], [7].

In the case of only a few dividend payments on the underlying equity, the original approach above - calculating a continuous dividend yield and using that in a closed form formula - is also no longer satisfactory, even for European options. The dividends occur at one or a few discrete times, but we are spreading them out throughout the life of the option by making this assumption, and this has a material effect on the stochastic process for the stock price.

Much theory has been developed to price (European) options under the assumption that the dividends are a known proportion of the stock price on the dividend payment date. See [8], for example. For European options with declared or predicted cash dividends, we will use the model of [7]. Another more practical and meaningful alternative is that the first (one or two) dividends are known or predicted in cash, whilst the remaining dividends are predicted as a proportion of stock price. In this case, we can combine the methods of [8] and [7].

In the case of an American call with one dividend, the formula of Roll, Geske, Whaley [9], [10], [11] is well known (amongst practitioners) to be arbitragable (and not so well known amongst software vendors, who often insist on offering this as the default model). Again, see [5], [7]. Furthermore, their approach does not allow for the pricing of American puts (as is well known, the pricing of American puts is in general more difficult than the pricing of calls).

Thus, for American options with a few dividends, we will use a finite difference scheme for pricing. This finite difference scheme easily accommodates the discrete jumps of dividends.

One can use the Lewis (European) or finite difference (European or American) approach for any number of dividends, if prepared to input them. As the number of dividends increases, the benefits of these approaches are outweighed by the superior speed of using the continuous dividend yield proxy in the Black-Scholes or Bjerksund-Stensland formula.

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For the finite difference schemes, we will use the Crank-Nicolson method for European options, and the projected Successive Over-Relaxation (SOR) implementation of the Crank-Nicolson Method [12, §9.4] for American options, and include the discretely-paid dividend corrections discussed in [13, §64.10.2] in either case.

The method of Lewis, Haug and Haug [7] for pricing European options where the underlying pays dividends is new.

As shown formally [7, Proposition 1], it is intuitive that the value of an option where the underlying has one dividend is equal at commencement to the present value of the option value immediately after the stock goes ex-dividend on the LDR. This ex-dividend value has a distribution given by the Black-Scholes value over the remaining interval and the evolution of the stock so far. We assume that the underlying is subject to volatility via a geometric brownian motion in the first time interval, the stock pays the discrete dividend, and then the residual stock value is again subject to the same volatility in the next time interval. This value is found as an 'impossible integral', using numerical integration techniques.

In the case of multiple dividends, we apply the process of [7, §4]. The idea is that the relevant integrals are of the Black-Scholes form, so we find a best-fit Black-Scholes formula for the price. This allows for iteration without resorting to multiple integrals.

On the PriceWorX input screen, one can choose between the following models:

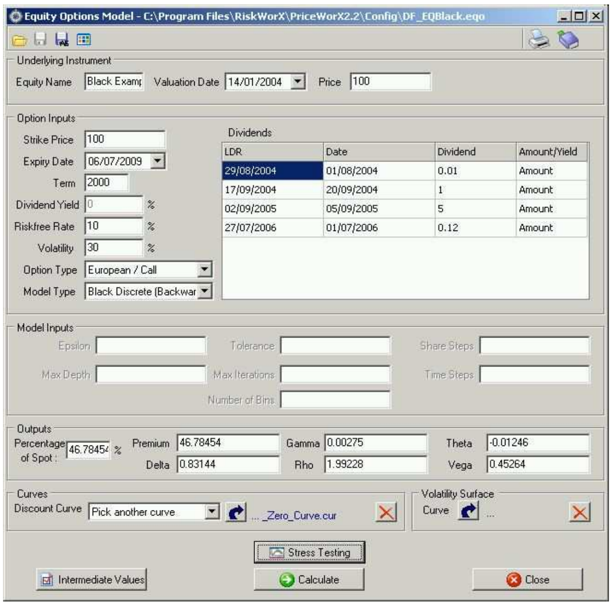

FIGURE 1. The input screen for equity options

- Black Discrete (Backwards) - for European equity options where the underlying has no or several dividends. A dividend yield is calculated consistent with this information and is use in the Black-Scholes formula.
- Bjerksund and Stensland method - for American equity options with the underlying having no or several dividends.
- Finite Differences - for European or American options with at least one but not many dividends, using finite differences.
- Analytic - the method of [7] for European options.
- Black DY - an old model retained for compatibility. The classic Black-Scholes type formula, with a dividend yield input.
- Black Discrete (Forward) - an old model retained for compatibility. Dividends are forward valued and added to the strike; (present valuing and subtracting from the spot is more common).

References

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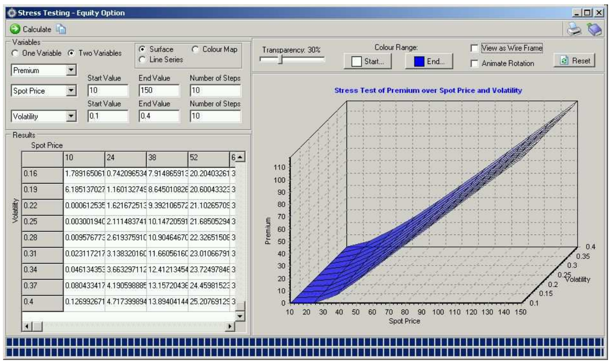

FIGURE 2. A matrix and display for profit and loss under stock and volatility movements

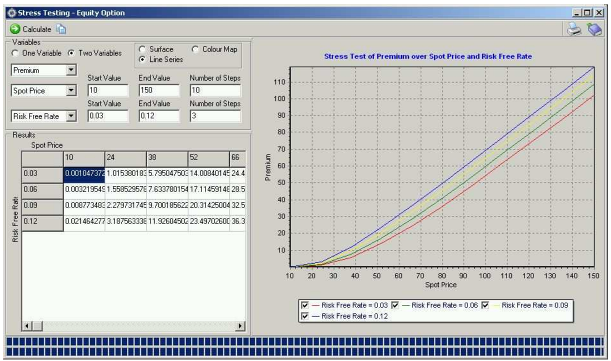

FIGURE 3. Profit and loss under different spot and interest rate scenario

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